Time-resolved magnetic sensing with electronic spins in diamond

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Sensing magnetic fields with quantum probes

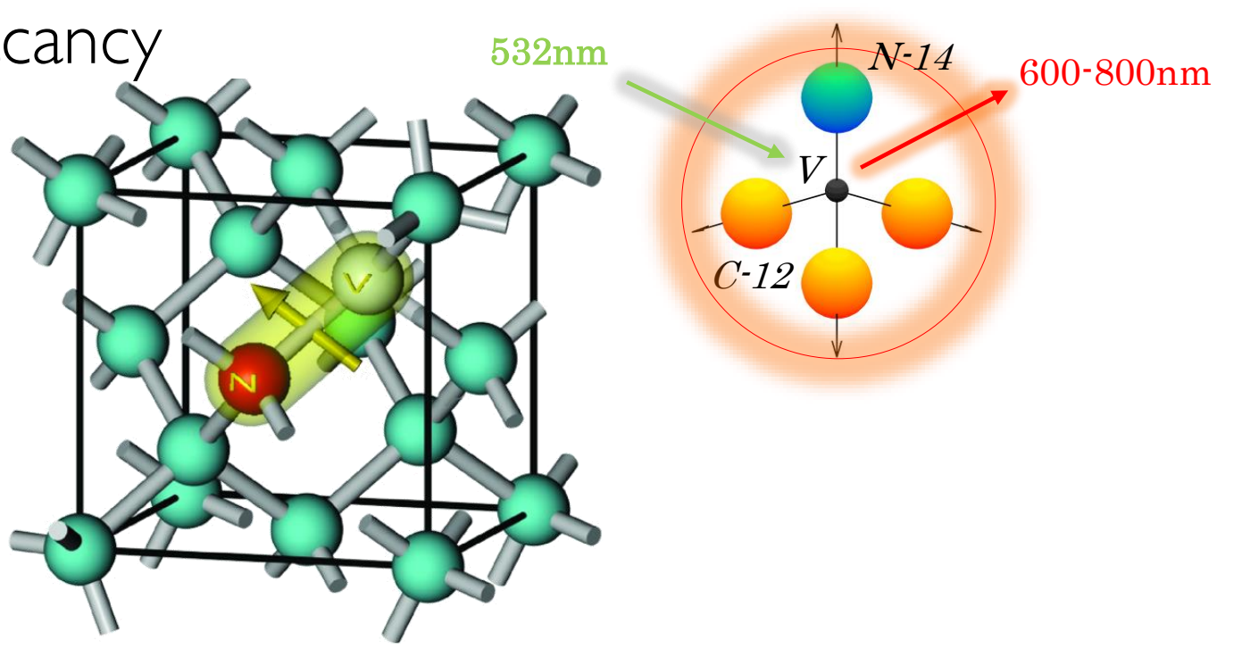
- Quantum probes sense magnetic fields with high sensitivity and spatial resolution.
- Systematic methods for measuring other than static or oscillating magnetic fields are needed.
- Spectral analysis in the Fourier domain suffers from drawbacks:
  - Quantum filters are naturally digital
  - Spectral reconstruction requires functional approximations or deconvolution algorithms.

Novel method for measuring time-varying magnetic fields

- We reconstruct the arbitrary temporal profile of time-varying fields using Walsh functions
- We characterize the performance of the method in term of reconstruction error and sensitivity
- We discuss applications to neuronal activity imaging

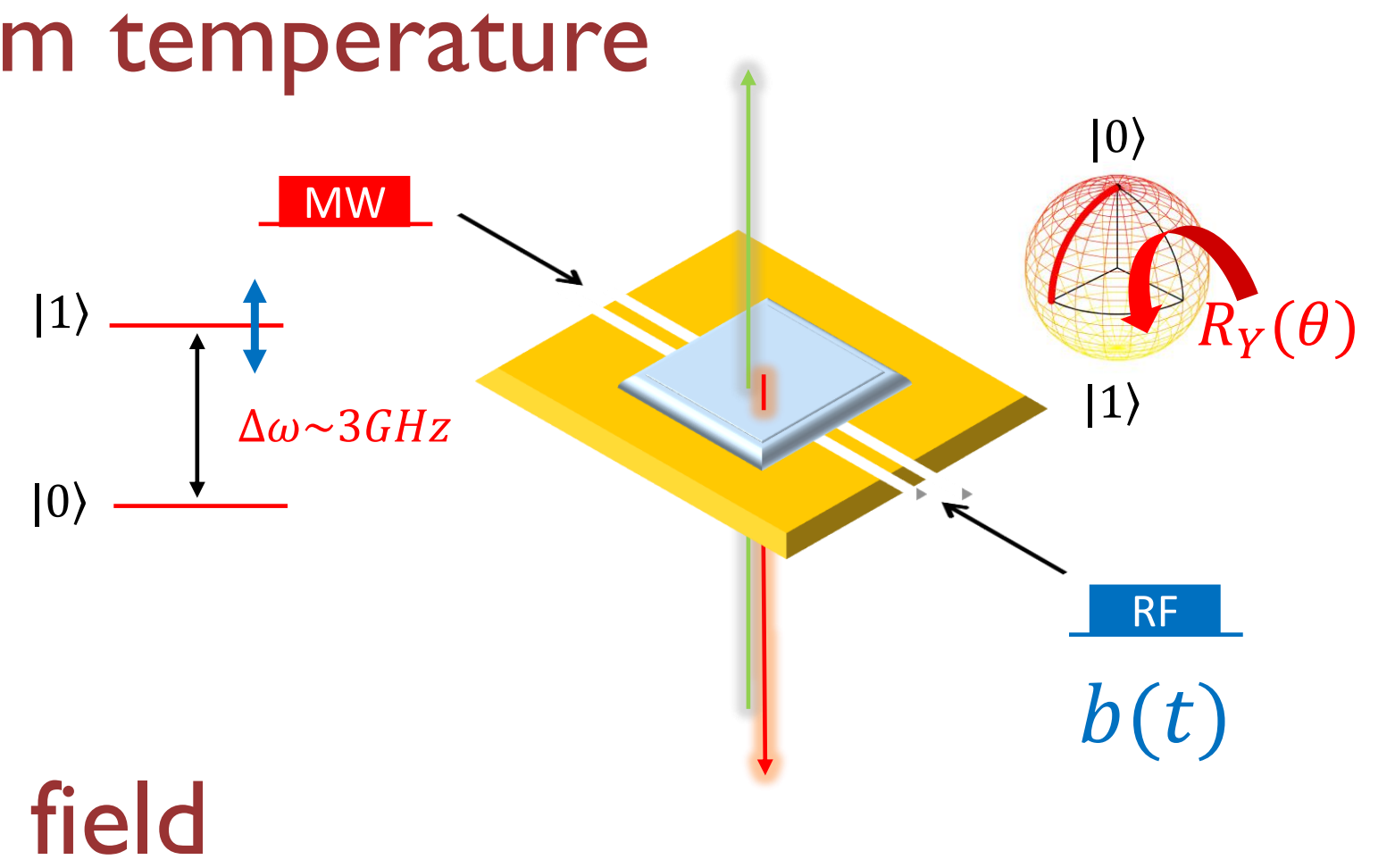
Our quantum probe is the nitrogen-vacancy center in diamond

- Color defect in the diamond lattice consisting of a vacancy adjacent to a substitutional nitrogen atom impurity
- Collection of emitted fluorescence (600-800 nm) via confocal microscopy
- Ground-state electronic spin-1 with good visibility and coherence properties at room temperature.



Control of spin properties at room temperature

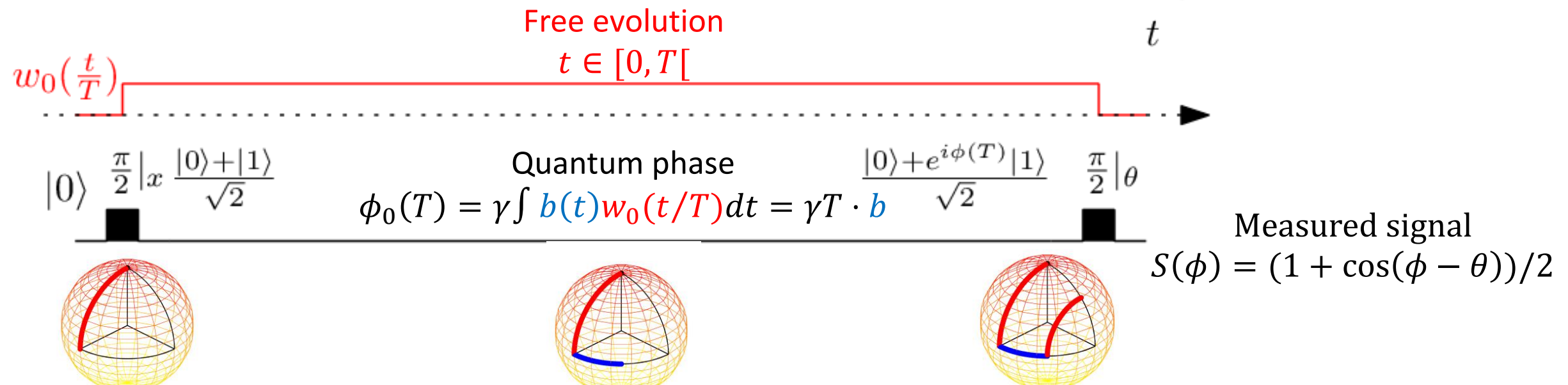
- Optical initialization and spin-state readout via confocal microscopy.
- Coherent control via coherent irradiation with microwave pulses.
- Applications in metrology and quantum information



Amplitude estimation of constant field

- Ramsey interferometry (d.c. magnetometry): measure the shift in the resonance frequency of a qubit interacting with an external field along its quantization axis.

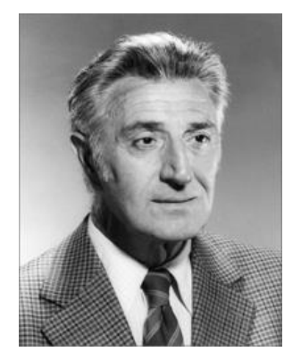
Norman F. Ramsey



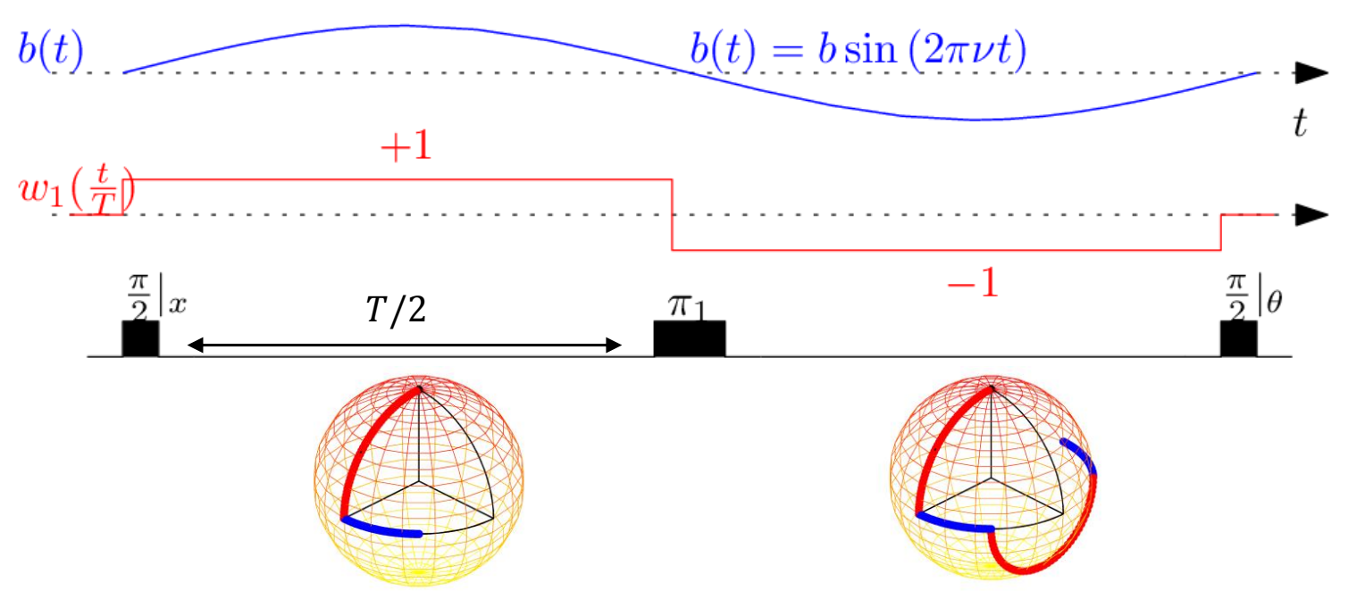
$$\boxed{\phi_0(T)/\gamma = T\cdot\frac{1}{T}\int_0^T b(t)w_0(t/T)dt = bT}$$

Amplitude estimation of oscillating field

- a.c. magnetometry: apply control π-pulses to synchronize the evolution of the qubit sensor with the time-varying field.



Erwin Hahn
1950

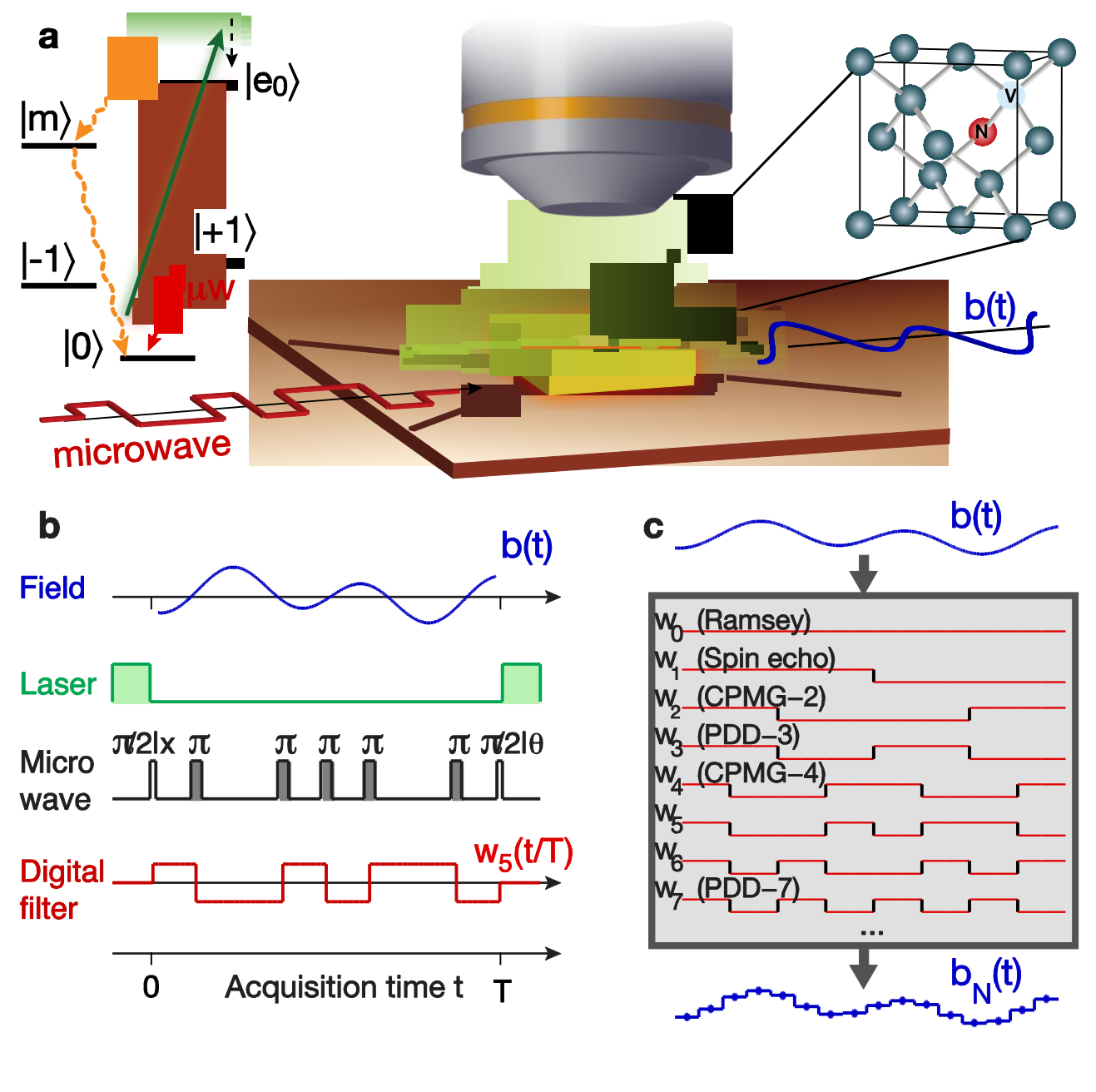


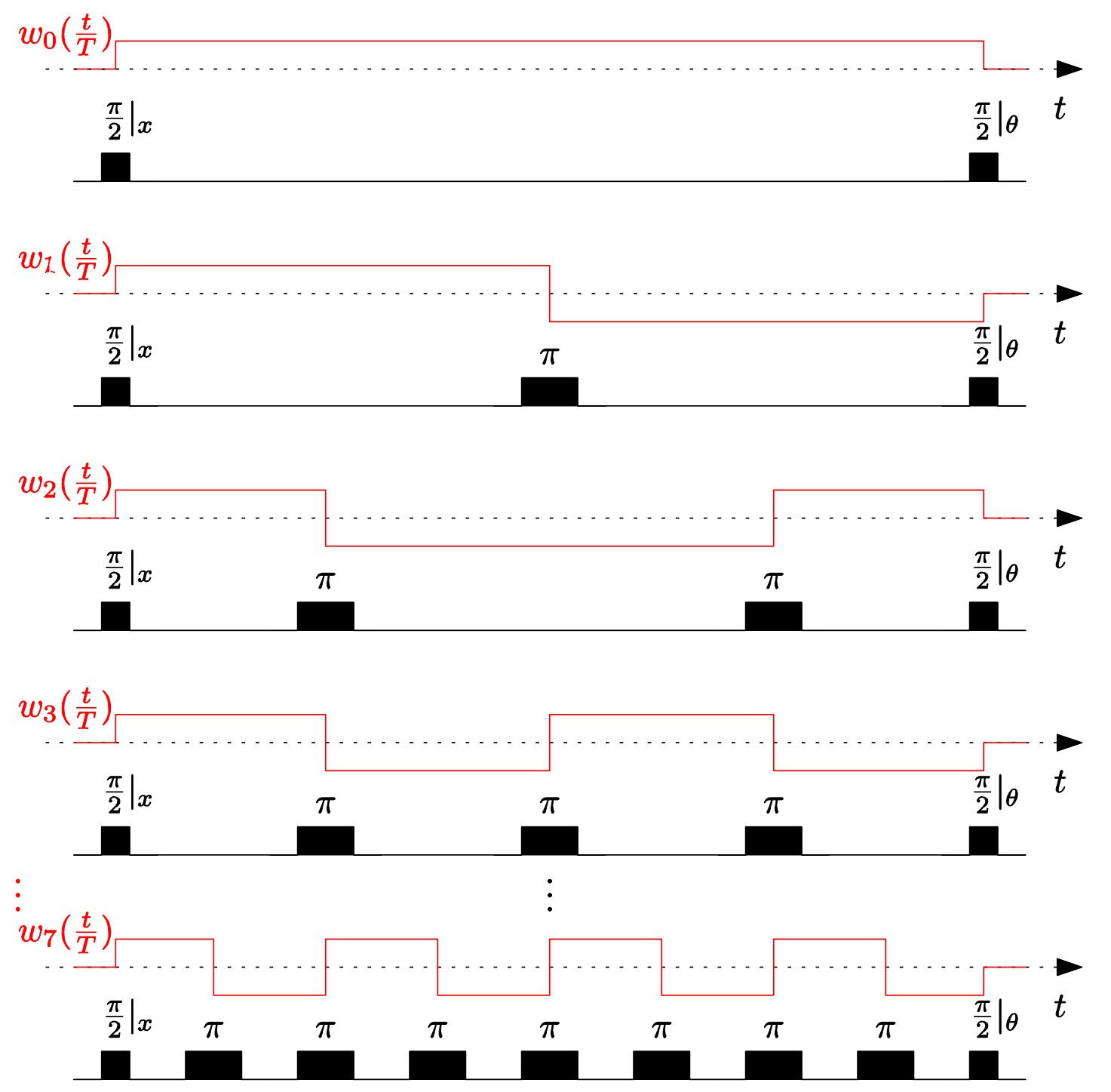
$$\phi_1(T)/\gamma = +\int_0^{T/2} b(t)dt - \int_{T/2}^{T} b(t)dt = bT\cdot\frac{2}{\pi}$$

$$\boxed{\phi_1(T)/\gamma = T\cdot\frac{1}{T}\int_0^T b(t)w_1(t/T)dt = bT\cdot\frac{1}{T}\int_0^T \sin(2\pi\nu t)w_1(t/T)dt}$$

Walsh reconstruction method [1, 2]

- Complete orthonormal basis of digital filters $\{w_m(t/T)\}$
- Contain known dynamical decoupling sequences (CPMG, PDD)
- Extract information while suppressing noise





(1) Modulate with *m-th Walsh sequence* $w_m(t/T)$ to extract the m*-th Walsh coefficient* $\hat{b}(m)$:

Walsh transform of $b(t)$ evaluated at *sequency* m:

$$\phi_m(T)/\gamma T = \frac{1}{T}\int_0^T b(t)w_m(t/T)dt = \hat{b}(m).$$

(2) Reconstruct the field from a set of N *Walsh coefficients* via *inverse Walsh transform*

$$b_N(t) = \sum_{m=0}^{N-1}\hat{b}(m)w_m(t/T).$$

Performance of the Walsh reconstruction method

- Bounded reconstruction error vanishing for finite number of coefficients

$$e_N = \|b_N(t) - b(t)\|_2 \le \max_{t\in[0,T]}\frac{|\partial_t b(t)|}{2^{n+1}} \text{ with } \lim_{N\to\infty} e_N = 0$$

- Quantifiable measurement sensitivity of the m-th Walsh sequence

$$\eta_m = \frac{v_m^{-1}}{\gamma_e C\sqrt{T}}\cdot\frac{1}{|\hat{f}(m)|} = \frac{\hat{\eta}(m)}{|\hat{f}(m)|}$$

The signal visibility is $v_m = \left(e^{-T/T_2(m)}\right)^{p(m)}$ with $T_2(m) > T_2$ (noise suppression)

- Trade-off between noise suppression and spectral information extraction

  $\hat{\eta}(m)$: Intrinsic sensitivity of the magnetometer in the presence of noise

  $|\hat{f}(m)|$: Walsh coefficient of the field measured with the m-th Walsh sequence

- Parameter estimation of time-varying fields or arbitrary waveform magnetometry (*a. w. magnetometry*) by choosing w_m that minimizes η_m.
- Quantifiable measurement sensitivity of the Walsh reconstruction method

$$\eta_N = \delta b_N\sqrt{MNT} = \sqrt{N\sum_m \eta_m^2} = \frac{\sqrt{N\sum_m v_m^{-2}}}{\gamma_e C\sqrt{T}\sqrt{N_{NV}}}$$

- Gain in sensitivity greater than $\sqrt{N}$ over sequential acquisition techniques (Ramsey)

$$\eta_{Walsh} < \frac{\eta_{Ramsey}}{\sqrt{N}}$$

Data compression and compressed sensing

- Reduction in total acquisition time by discarding negligible coefficients [2]
- Quantifiable increase in reconstruction error due to truncation of few coefficients
- Compressed sensing for S-sparse signals provides logarithmic scaling in resources [3]

$$N \to S\log_2(N)$$

Reconstruction of monochromatic fields

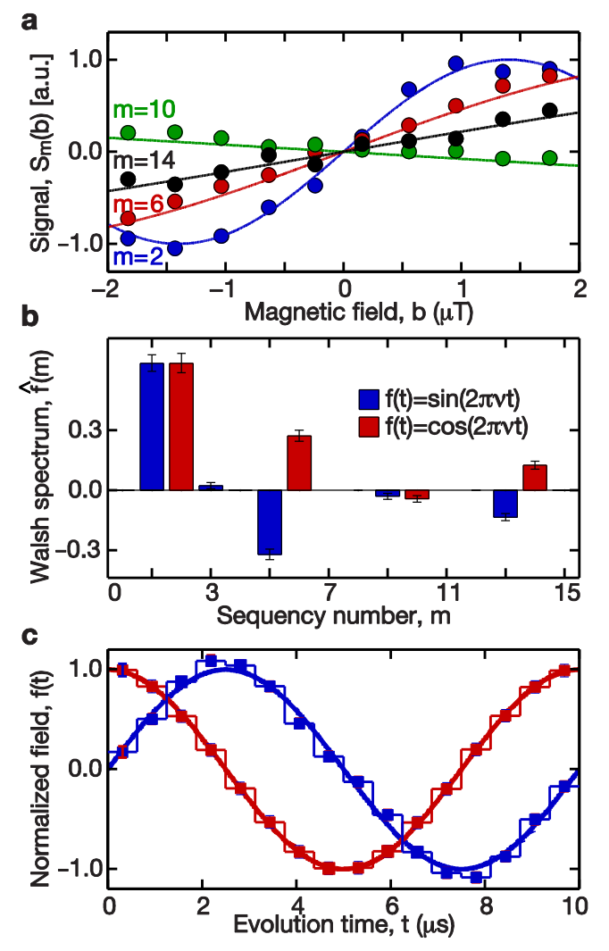
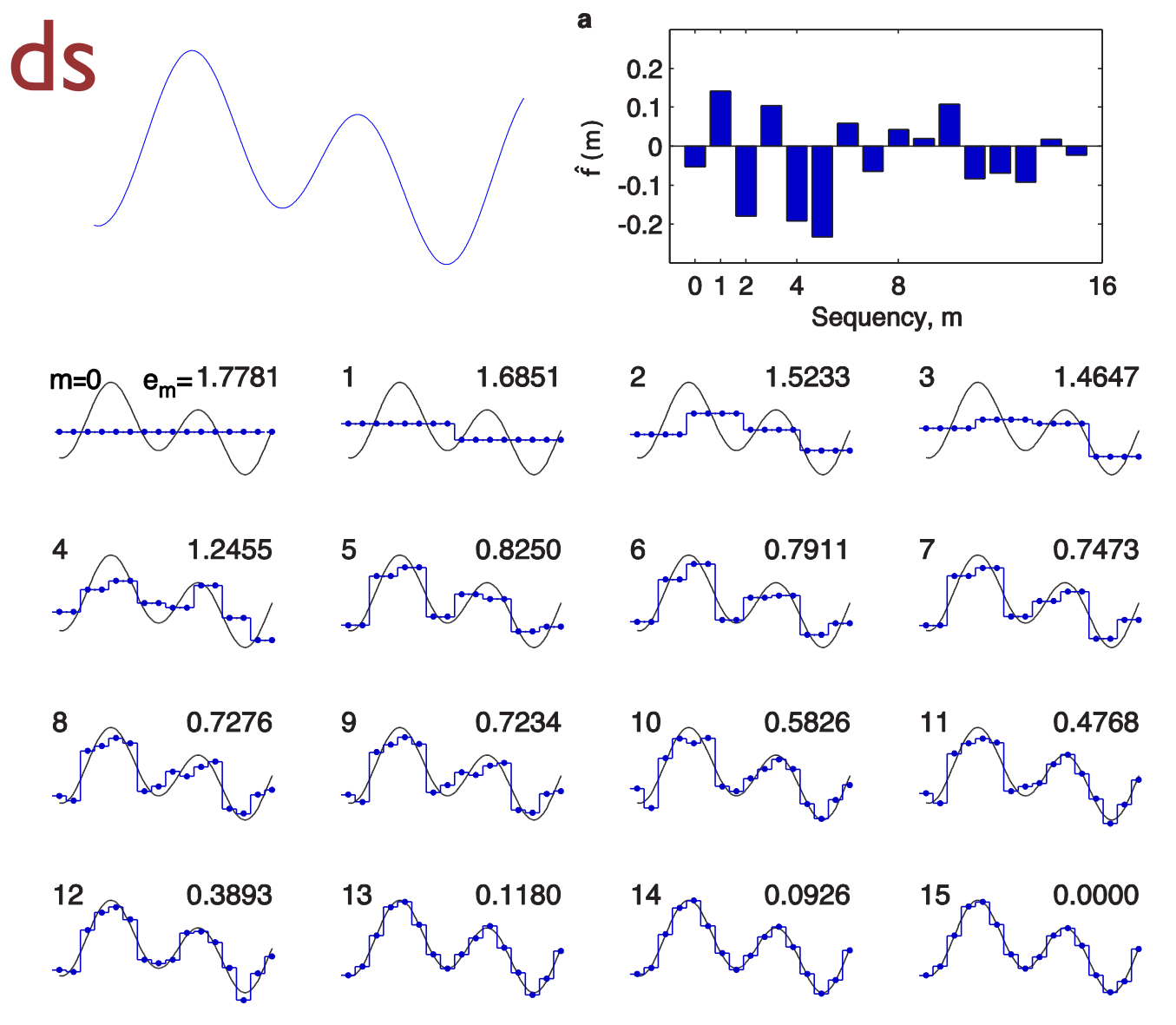


Figure 1 | 16-point reconstruction of a **ν**= 100 kHz sine and cosine waveform in the time-domain (T=10**μ**s). **a**, Measured signal with different Walsh filters for a sine and cosine waveform. The slope is proportional to the m-th Walsh coefficient. **b**, Measured Walsh spectrum up to m=16. **c**, the reconstructed field (filled squares) is a 16-point piecewise-constant approximation to the expected field (solid line).



Reconstruction of arbitrary polychromatic fields

- Walsh transform is linear, i.e., $b(t) = \sum_j \sin(2\pi\nu_j t + \alpha_j) \Rightarrow \hat{b}(m) = \sum_j \hat{b}_j(m)$.
- Walsh method outperforms reconstruction with incomplete sets of filters (CPMG, PDD).

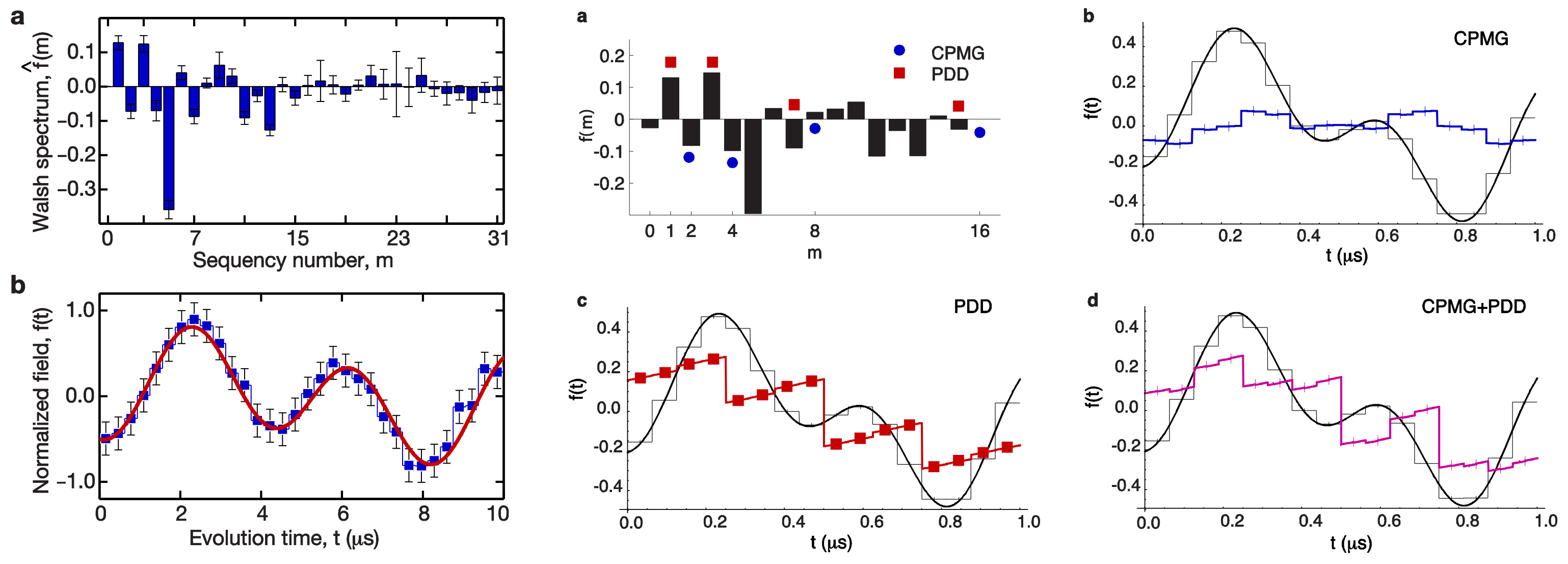


Figure 2 | 32-point reconstruction of a bichromatic field. **a,** Measured Walsh spectrum up to fifth order (N=2^5). **b**, The reconstructed field (filled squares) is a 32-point approximation to the expected field (solid line, not a fit). **c**, The Walsh method outperforms the reconstruction with incomplete sets of filters such as PDD and CPMG sequences.

Reconstruction of simulated neuronal action potential fields

- Simulated action potential (NEURON) of a rat hippocampal mossy fiber bouton approximated by a skew normal impulse.

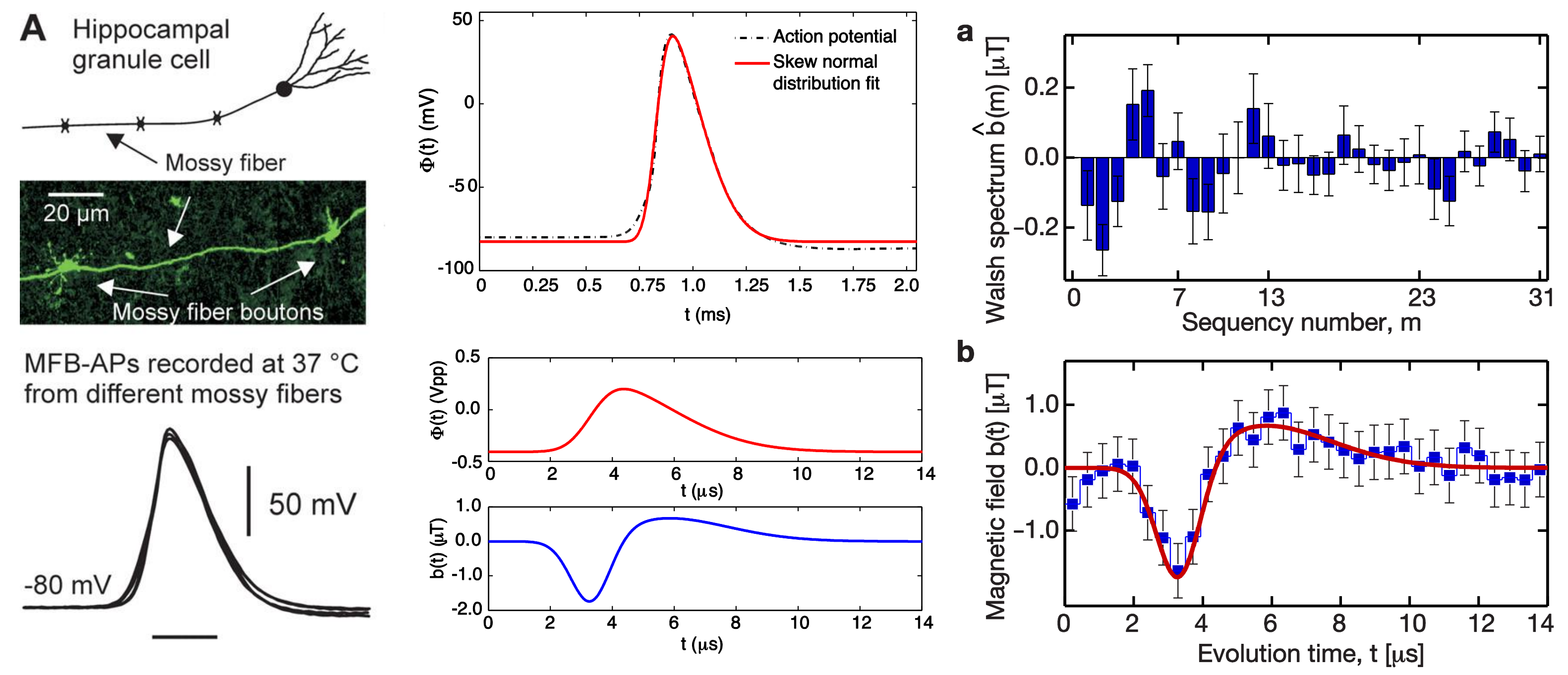


Figure 3 | 32-point reconstruction of a magnetic field radiated by a skewed normal impulse flowing through the physical model of a neuron. **a,** Measured Walsh spectrum up to fifth order (N=2^5). The Walsh coefficients were obtained by fixing the amplitude of the field and sweeping the phase of the last read-out pulse. **b.** The reconstructed field (filled squares) is a 32-point approximation to the expected field (solid line, not a fit).

References

[1] Cooper, A., Magesan, E., Yum, H.N., Cappellaro, P. Time-resolved magnetic sensing with electronic spins in diamond. *arXiv*:1305.6082 (2013).
[2] Magesan, E., Cooper, A., Yum, H.N., Cappellaro, P. Reconstructing the profile of time-varying magnetic fields with quantum sensors. *Phys. Rev. A* **88**, 032107 (2013).
[3] Magesan, E., Cooper, A., Cappellaro, P. Compressing measurements in quantum dynamic parameter estimation. *arXiv*:1308.0313 (2013).