Realizing the Harper Hamiltonian with Spin Mixtures of Ultracold Atoms

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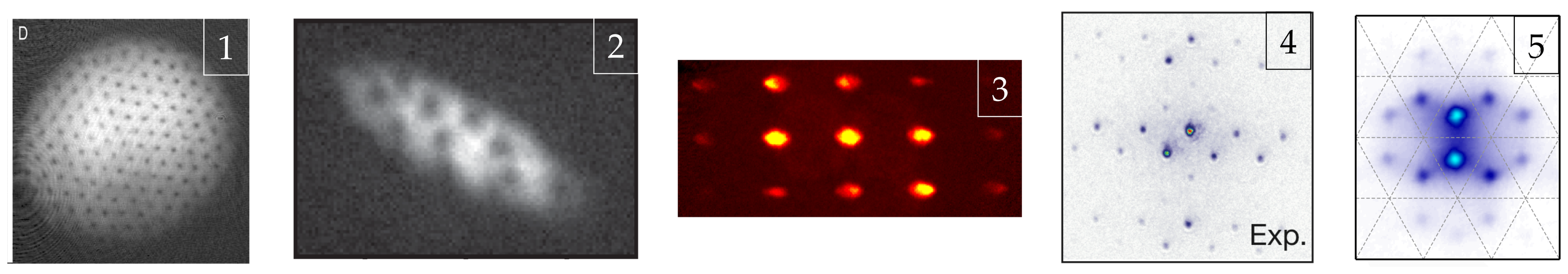
Motivation

Interesting states of matter (e.g. integer and fractional quantum Hall states, topological insulators, etc.) arise in systems of interacting, *charged* particles in a magnetic field. Can such states be realized in a well-controlled, defect free environment with a *neutral* quantum gas?



Much work has been done on realizing effective magnetic fields - in a bulk gas [1,2] and on a lattice as well [3-5].

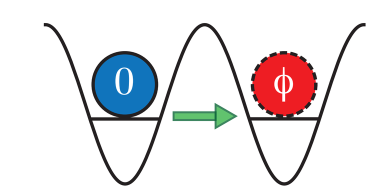
Realizing a system with a large ratio of flux quanta to particle number remains an open question addressed by this work [6,7] and work in Munich [8]

The Harper Hamiltonian

Model of *charged* particles on a lattice in a magnetic field [9]:

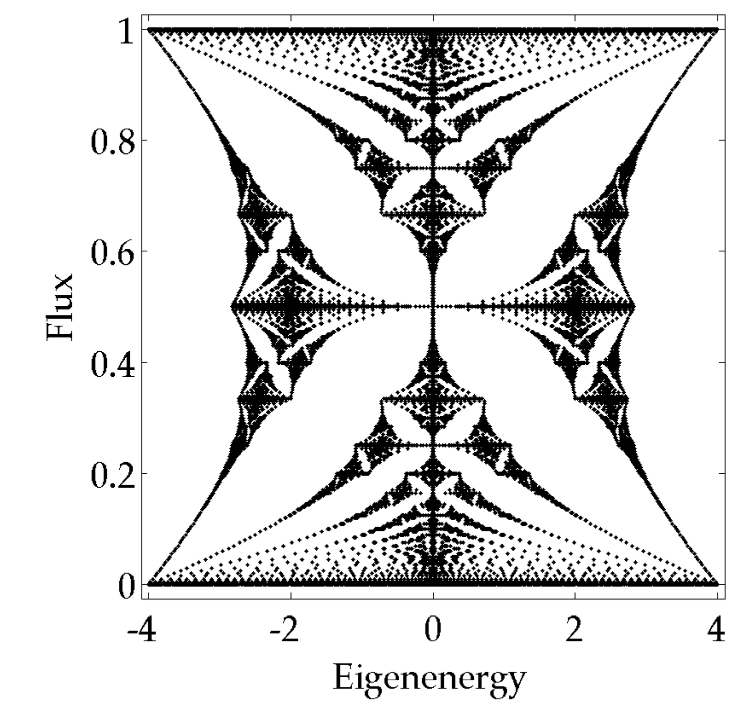
$$H = -\sum_{\langle m,n \rangle} \left(K e^{i\phi_{m,n}} \hat{a}^\dagger_{m,n} \hat{a}_{m,n} + h.c. \right)$$

$$\phi_{m,n} = \int_{\mathbf{R}_m}^{\mathbf{R}_n} q\mathbf{A}\cdot\mathbf{ds}/\hbar$$



Spectrum is fractal and is known as Hofstadter's Butterfly [10]:

On a ~1 A lattice, magnetic fields of ~10,000 T are necessary for flux densities of order 1



The band structure is topologically non-trivial, e.g. for a=1/2 the ground band has Chern number 1.

The Main Idea for Uniform Flux

A uniformly tilted lattice suppresses normal tunneling in the x-direction

Resonant tunneling is re-established using a pair of far detuned Raman lasers, imprinting a phase on each link with the y-momentum transfer

$$K = \frac{\Omega}{2}\int d^2\mathbf{r}\, w^*(\mathbf{r}-\mathbf{R})e^{\pm i\delta\mathbf{k}\cdot\mathbf{r}} w(\mathbf{r}-\mathbf{R}_{m,n} \mp a\hat{\mathbf{x}})$$
$$= K_0 e^{\pm i\phi_{m,n}}$$

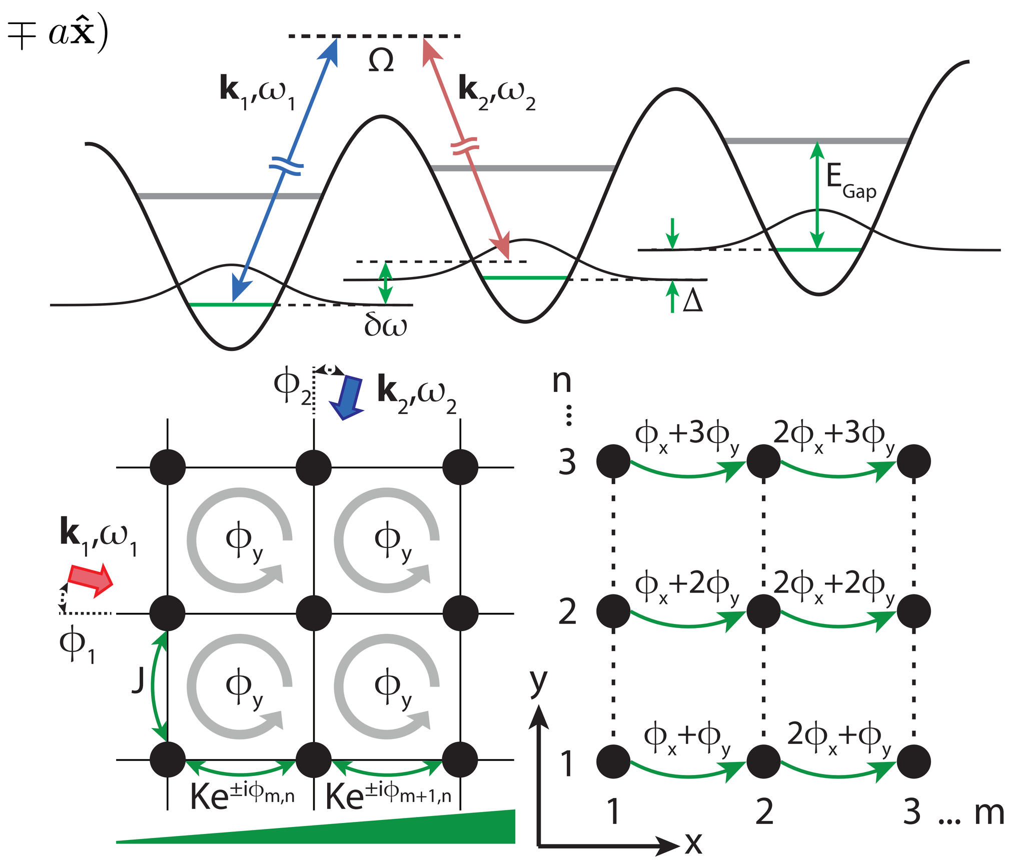
Both x- and y- momentum needed!

$$K_0 = \frac{\Omega}{2}\int dx\, w^*(x)e^{\pm ik_x x}w(x-a)\ldots$$
$$\ldots\times\int dy\, w^*(y)e^{\pm ik_y y}w(y)$$

The momentum defines the gauge

$$\phi_{m,n} = mk_x a + nk_y a$$

$$\mathbf{A} = \frac{\hbar}{a}(k_x x + k_y y)\,\hat{\mathbf{x}}$$



Emulating the Quantum Spin Hall Effect

Uniform energy offset can be made with magnetic field gradient, so different magnetic moments see different tilts

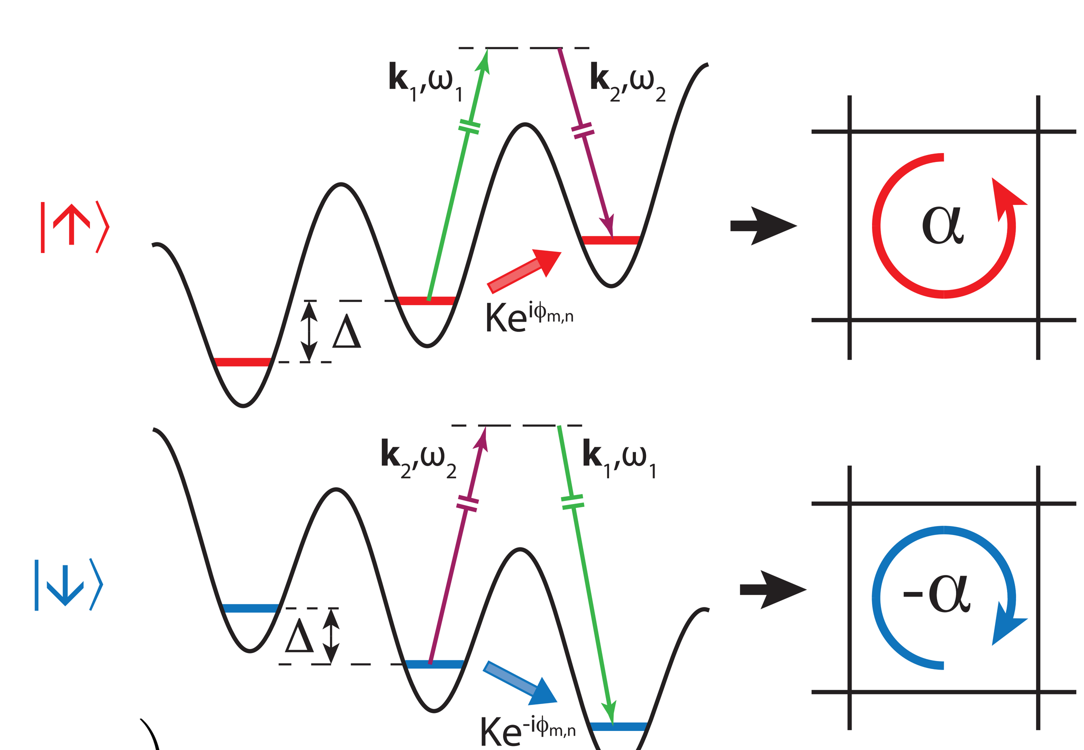
Changing direction of the momentum transfer changes the sign of the gauge potential:

$$\mathbf{A} = \frac{\hbar}{a}(k_x x + k_y y)\,\hat{\mathbf{x}}\,\hat{\sigma}_z$$

This realizes an Abelian form of spin-orbit coupling that *does not* require near resonant lasers.

It can be expressed by a term in the Hamiltonian:

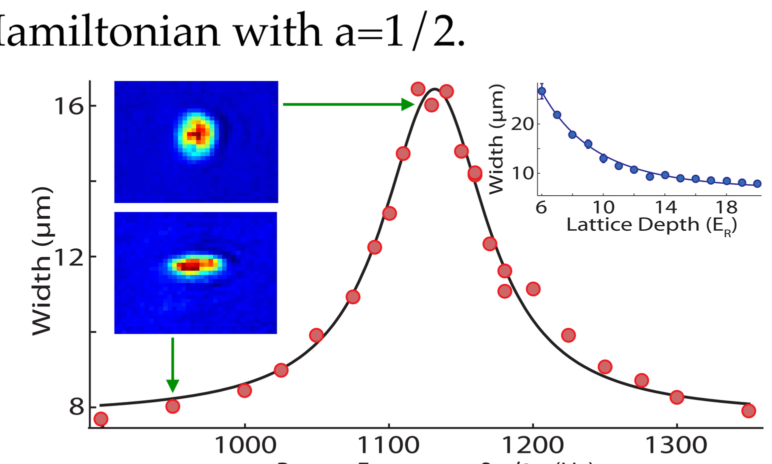
$$H_{SOC} = \frac{\hbar}{2ma}\Big((2p_x x - i\hbar)k_x + 2p_x k_y y \Big)\hat{\sigma_z}$$



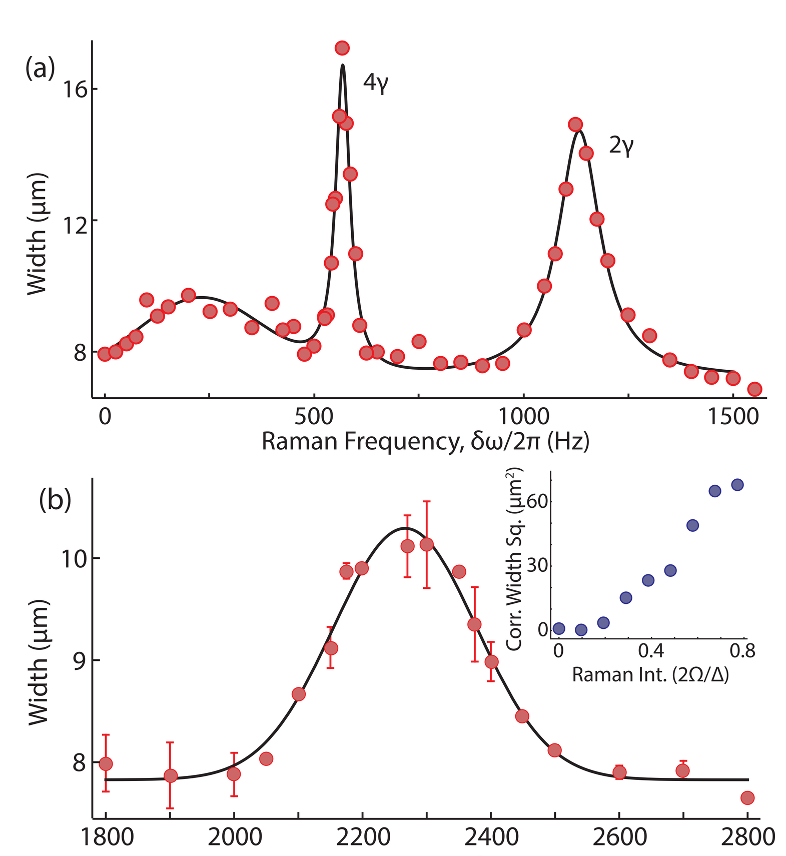
Each spin independently realizes a quantum Hall state with opposite Chern numbers. The total spin is conserved, as such the system is protected by a $\mathbb{Z}$ topological index.

Experimental Realization

Tunneling in the tilt direction can only happen on Raman resonance, so an *in situ* expansion measurement reveals the presence of tunneling. We realize the Hamiltonian with a=1/2.



Observation of laser-assisted tunneling!



Four-photon, nearest-neighbor and two-photon next-nearest -neighbor tunneling

The tunneling rate scales non-linearly with the Raman drive, and can be exactly derived in the mapping from the Wannier-Stark problem to the Harper Hamiltonian. We observe qualitative agreement.

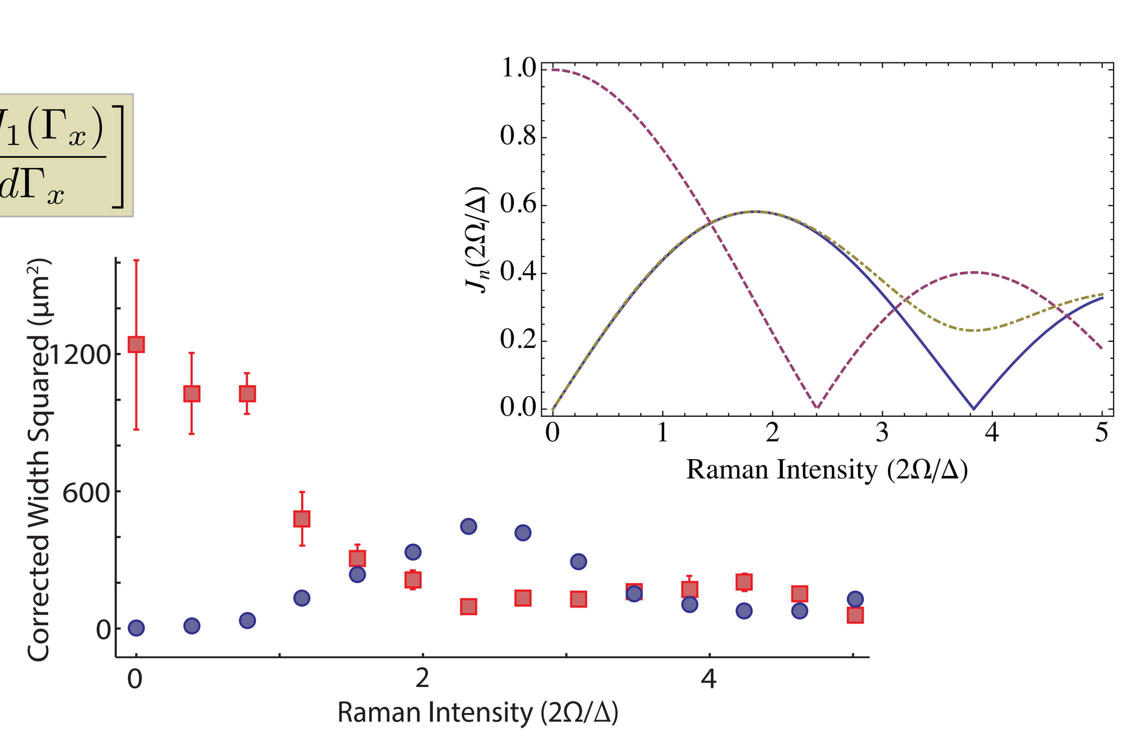
$$K = \Omega\Phi_{y0}\left[\Phi_{x1}\frac{J_1(\Gamma_x)}{\Gamma_x} + i\Phi'_{x1}\frac{dJ_1(\Gamma_x)}{d\Gamma_x}\right]$$

$$J = J_y J_0(\Gamma_x)$$

$$\Gamma_i = \frac{2\Omega\Phi_{y0}\Phi_{x0}}{\Delta}\sin\left(\frac{k_i a}{2}\right)$$

In the tight-binding limit:

$$J = J_y J_0\left(\frac{2\Omega}{\Delta}\right) \qquad K = J_x J_1\left(\frac{2\Omega}{\Delta}\right)$$



Future Directions

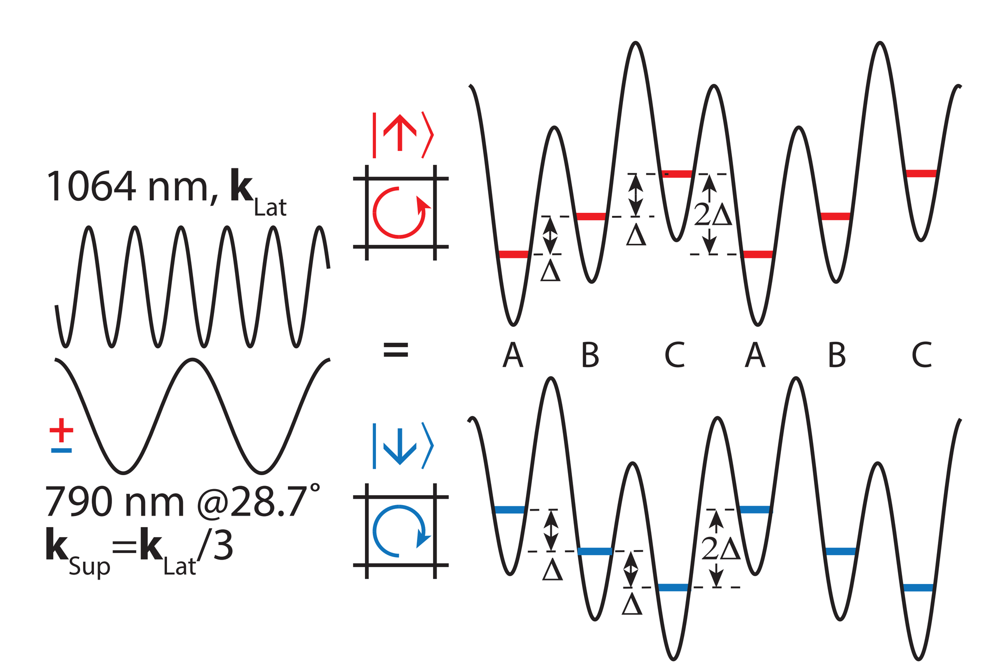
Find the ground state of the system: Hofstadter's Butterfly

Superfluid dominated by next-nearest-neighbor interactions

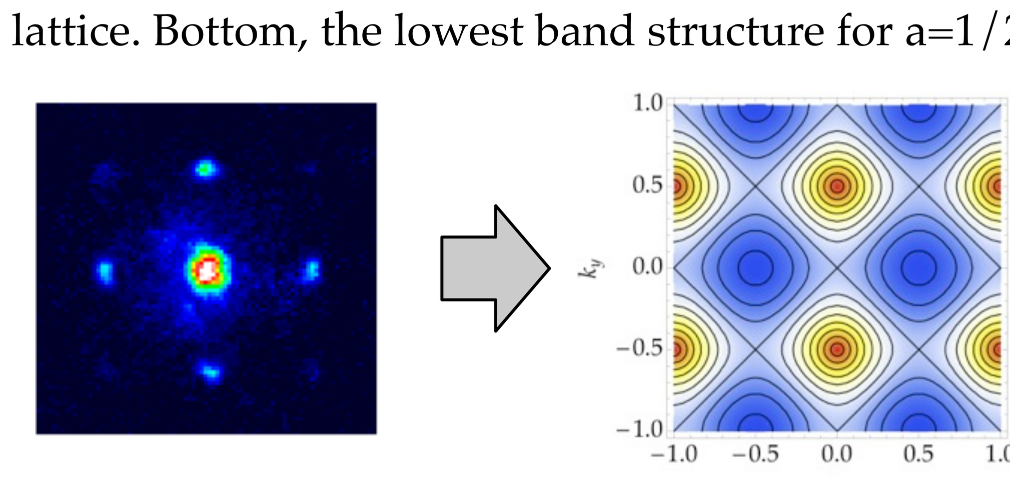
Couple spin states with microwave drive. If time-reversal symmetry is preserved, the system should exhibit $\mathbb{Z}_2$ topological index [11,12].

Add interactions with a confining lattice in the 3rd dimension. Near the Mott transition, the system may exhibit bosonic Laughlin states [13].

With interactions, possible spin drag measurements [14], interaction induced cyclotron motion, and much more!



Top, superlattice implementation of the tilted lattice. Bottom, the lowest band structure for a=1/2.



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